Mesh–Refinement Strategies for Optimal Control of Kite Power Systems

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Luís Tiago Paiva[1] and **Fernando A.C.C. Fontes[2]**

SYSTEC–ISR, Faculdade de Engenharia, Universidade do Porto, Portugal

[1]ltpaiva@fe.up.pt , [2]faf@fe.up.pt

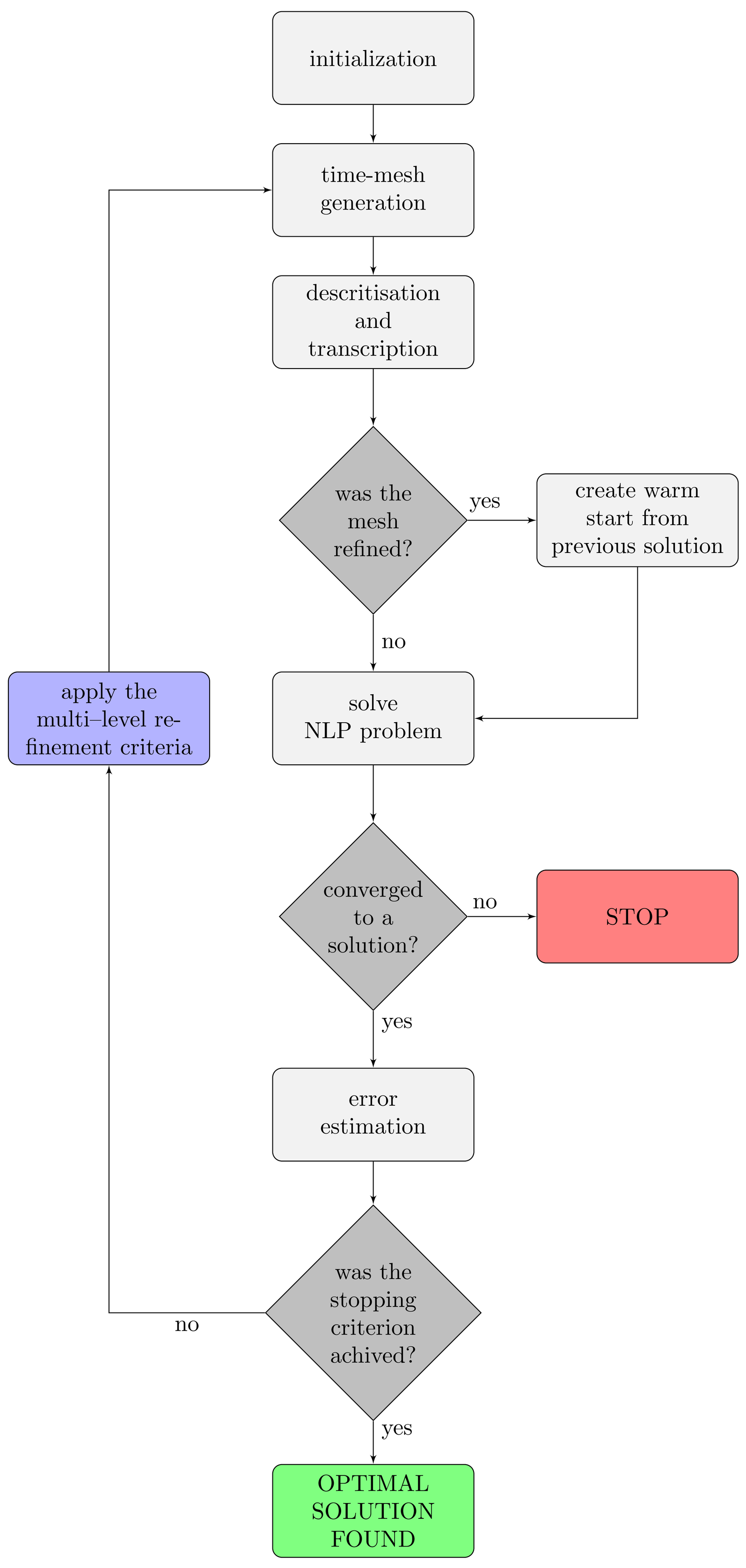
U. PORTO

Time–mesh refinement

Continuous time optimal control problem, in Bolza form, with input and state constraints:

$$\begin{aligned} \text{Minimize } & \int_{t_0}^{t_f} \mathrm{L}(t,\mathbf{x}(t),\mathbf{u}(t))\,\mathrm{d}t + \mathrm{G}(t_0,\mathbf{x}(t_0),t_f,\mathbf{x}(t_f)) & \\ \text{subject to } & \dot{\mathbf{x}}(t) = \mathbf{f}(t,\mathbf{x}(t),\mathbf{u}(t)) & \text{a.e. } t \in [t_0,t_f] \\ & \mathbf{u} \in \mathbb{U}(t) \subset \mathbb{R}^m & \text{a.e. } t \in [t_0,t_f] \\ & \mathbf{x}(t) \in \mathbb{X} & \forall t \in [t_0,t_f] \\ & (\mathbf{x}(t_0),\mathbf{x}(t_f)) \in \mathbb{X}_{01} & \end{aligned}$$

Algorithm



Refinement strategy

The original coarse mesh is divided in K mesh intervals

$$\mathscr{S}_k = [\tau_{k-1}, \tau_k[,\ k = 1,\ldots,K-1 \quad \text{and} \quad \mathscr{S}_K = [\tau_{K-1}, \tau_K]$$

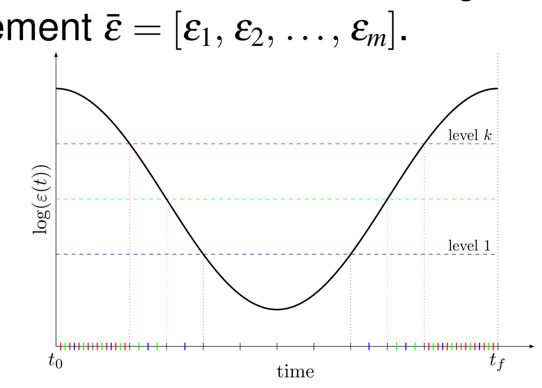
where $\tau_0 < \tau_1 < \ldots < \tau_K$ coincide with nodes.

The mesh intervals $\mathscr{S}_k$ form a partition of the time interval,

$$\bigcup_{k=1}^{K} \mathscr{S}_k = [t_0, t_f] \quad \text{and} \quad \bigcap_{k=1}^{K} \mathscr{S}_k = \emptyset .$$

After selecting the intervals $\mathscr{S}_k$ that verify the refinement criteria, they are divided into smaller subintervals according to the user–defined levels of refinement $\bar{\varepsilon} = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m]$.

A subinterval $\mathscr{S}_{k,i}$ is at the i^{th} level of refinement if $\mathscr{S}_{k,i} \in [\varepsilon_i, \varepsilon_{i+1}[$, for $i = 1,\ldots,m$, and it will be refined by adding $\mathscr{N}^i$ of equidistant nodes between each two mesh points.



In our case, we define 3 refinement criteria:

- relative error of the trajectory (state variables) ($\varepsilon_{\mathbf{x}}$)
- relative error of the adjoint multipliers ($\mathbf{q}$ multipliers) ($\varepsilon_{\mathbf{q}}$)
- a combination of both criteria

and we consider a threshold for the relative error of the trajectory as the stopping criterion.

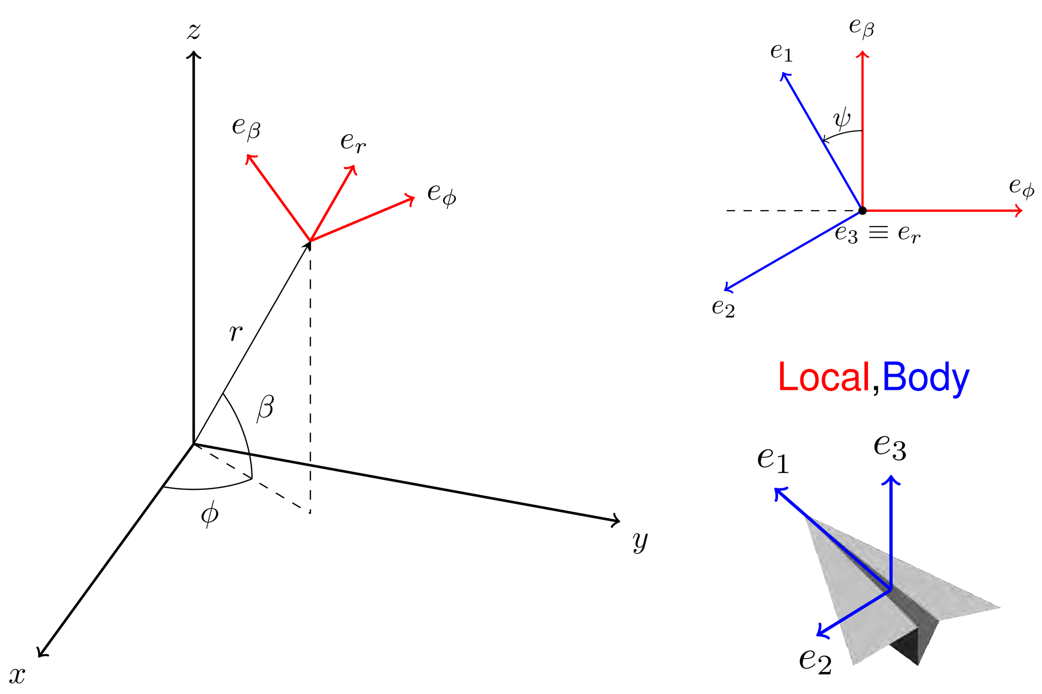
Relationship between multipliers: OCP and NLP

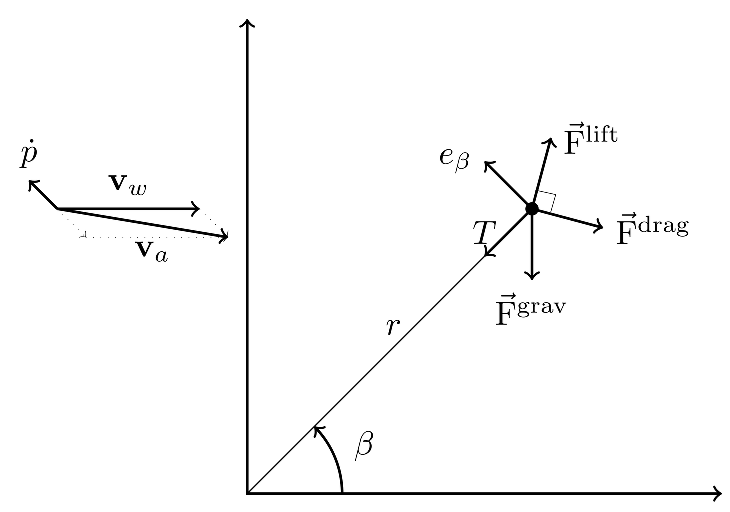
$$\begin{array}{lllll} & \nearrow \text{Max. Principle} & \to q_{MP} & & \\ \text{Continuous OCP} \to \text{Discrete OCP} & \to \text{Discrete MP} & & \to q_{DMP} & \\ & \searrow \text{Transcribe NLP} & \to \text{KT conditions} & \to q_{KT} & \end{array}$$

- q_{DMP} and q_{KT} coincide
- q_{DMP} is a discrete aproximation of q_{MP}
- q_{KT} is an output of the NLP solver

Kite power system

Coordinate Systems and Forces





Forces

Optimal Control Problem

Consider $\mathbf{x} = \left(r, \phi, \beta, \psi, \dot{\phi}, \dot{\beta}\right)$ and $\mathbf{u} = (v_t, \alpha, \omega)$:

$$\begin{aligned} \text{Maximize } & \int_0^{t_f} \dot{r}(-T)\,\mathrm{d}t & \\ \text{subject to } & & \\ & \dot{\mathbf{x}}(t) = f(\mathbf{x}(t),\mathbf{u}(t)) & \text{a.e. } t \in [0,t_f] \\ & \mathbf{x}(0) = \mathbf{x}_0 & \\ & \mathbf{x}_{f,\min} \le \mathbf{x}(t_f) \le \mathbf{x}_{f,\max} & \\ & \mathbf{x}_{\min} \le \mathbf{x}(t) \le \mathbf{x}_{\max} & \forall t \in [0,t_f] \\ & \mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max} & \text{a.e. } t \in [0,t_f] \end{aligned}$$

where

$$f(\mathbf{x}(t),\mathbf{u}(t)) = \begin{bmatrix} v_t \\ \dot{\phi} \\ \dot{\beta} \\ \omega \\ (F_\phi^{aer}(\alpha) + F_\phi^{iner})/(mr\cos(\beta)) \\ (F_\beta^{aer}(\alpha) + F_\beta^{grav} + F_\beta^{iner})/(mr) \end{bmatrix},$$

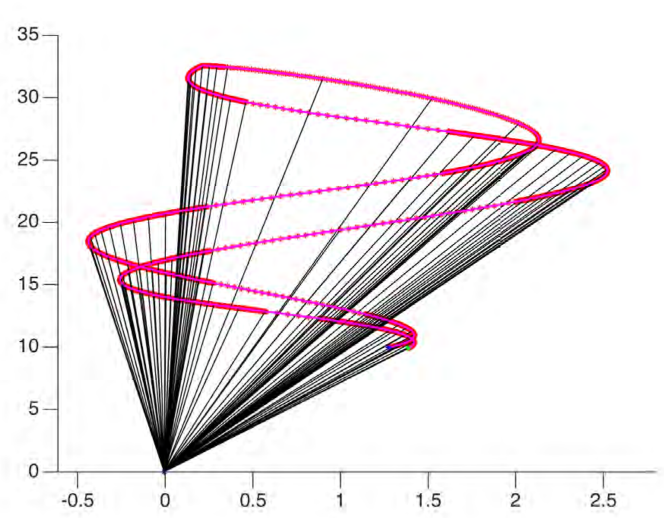
$$-T = F_r^{aer}(\alpha) + F_r^{grav} + F_r^{iner} .$$

Numerical results

The proposed algorithm is implemented in MATLAB using ICLOCS and IPOPT. The problem is solved using two meshes:

- π_{ML} mesh generate by the adaptive refinement strategy
- π_{F} equidistant spacing mesh considering the lowest Δt of π_{ML}

2D Problem

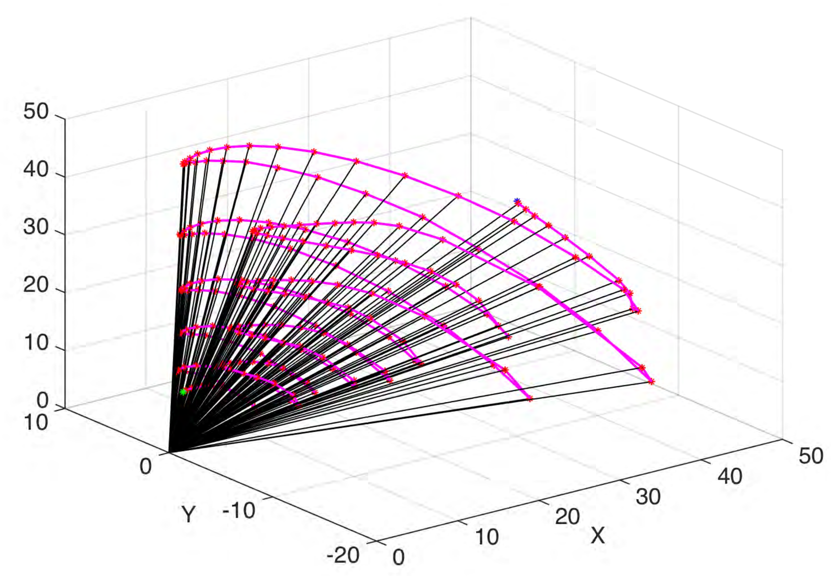


2D Optimal Trajectory

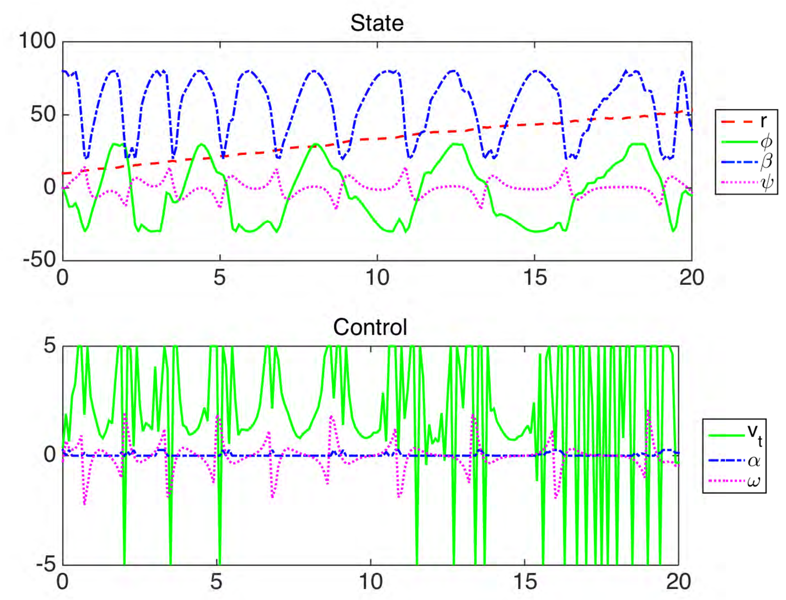
π_j	N_j	Δt_j	I_j	Objective	$\left\|\left\|\varepsilon_{\mathbf{x}}^{(j)}\right\|\right\|_\infty$	CPU time (s) Solve	$\varepsilon_{\mathbf{x}}$
π_0	51	1/50	253	53.216968	1.795E−2	4.754	0.107
π_1	474	1/500	122	54.156063	2.328E−3	4.722	0.856
π_2	2877	1/5000	26	54.266839	3.168E−4	6.107	3.383
π_{ML}	2877	1/5000	401	54.266839	3.168E−4	15.583	4.346
π_{F}	5001	1/5000	1342	54.266873	2.201E−4	431.234	10.913

Mesh comparison

3D Problem



Optimal Trajectory



State and Control

Concluding remarks

Main advantages of refinement strategies

- no need to define *a priori* the most appropriately mesh,
- local mesh resolution only where it's required,
- first solution obtained after few iterations since an initial coarse mesh is used,
- warm–start after refinement leads to faster re–solve,
- less nodes in the overall procedure results in significant savings in memory and computational cost.

... of our refinement algorithm

- multi–level refinement leads to faster convergence,
- local error on $\mathbf{q}$ gives a more accurate/faster error estimate,
- a solution is obtained very quickly (if the procedure ends in an early stage).

... in kite power systems

- much faster in the 2D problem,
- optimal solution found without informative initial guess in the 3D problem.

References

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