Mesh–Refinement Strategies for Optimal Control of Kite Power Systems

AIRBORNE WIND ENERGY CONFERENCE 2015

15–16 June, Delft

Luís Tiago Paiva[1] and **Fernando A.C.C. Fontes[2]**

SYSTEC–ISR, Faculdade de Engenharia, Universidade do Porto, Portugal

[1]ltpaiva@fe.up.pt , [2]faf@fe.up.pt

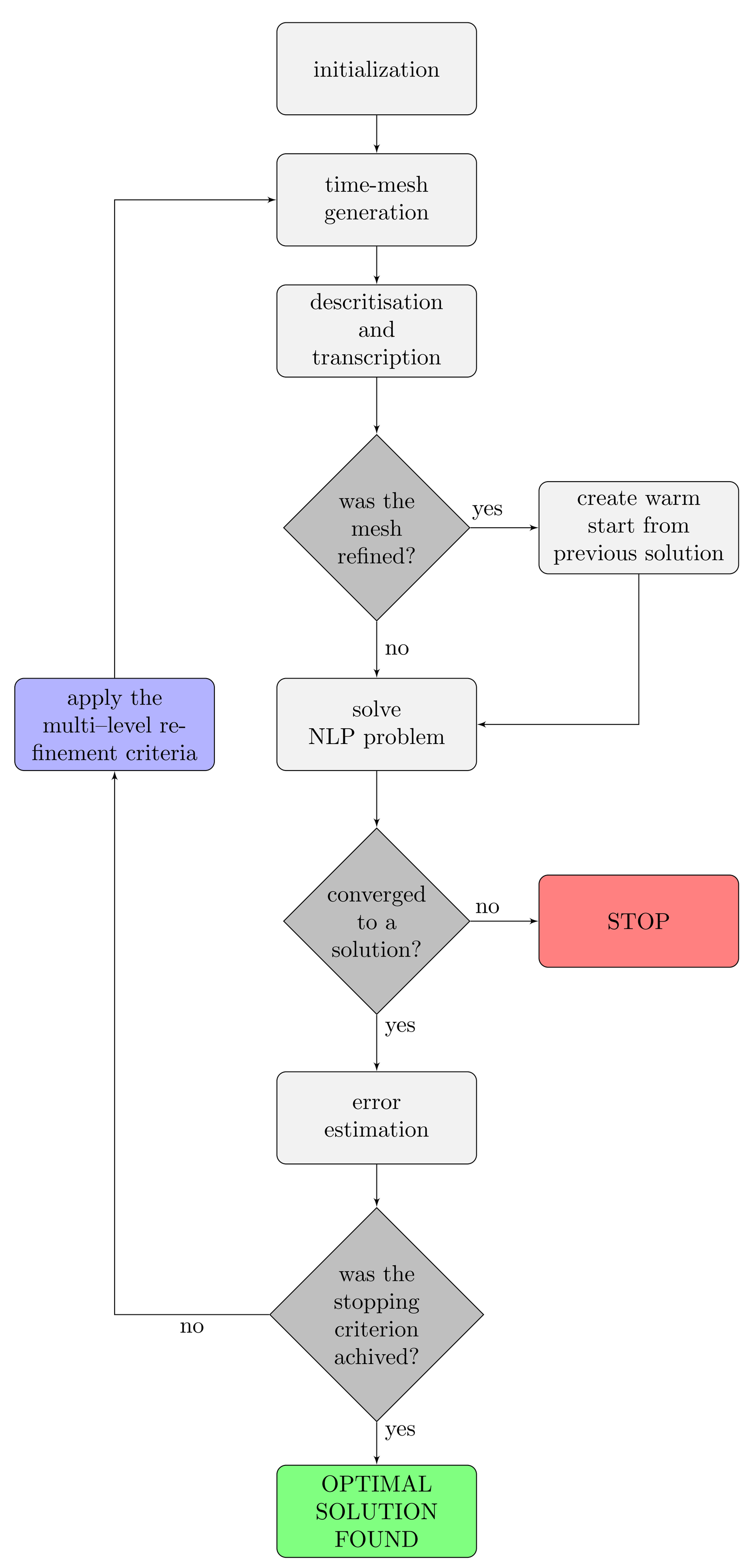
U.PORTO

Time–mesh refinement

Continuous time optimal control problem, in Bolza form, with input and state constraints:

$$
\begin{aligned}
&\text{Minimize} \int_{t_0}^{t_f} \mathrm{L}(t,\mathbf{x}(t),\mathbf{u}(t))\,\mathrm{d}t + \mathrm{G}(t_0,\mathbf{x}(t_0),t_f,\mathbf{x}(t_f)) \\
&\text{subject to } \dot{\mathbf{x}}(t) = \mathbf{f}(t,\mathbf{x}(t),\mathbf{u}(t)) && \text{a.e. } t\in[t_0,t_f] \\
&\qquad \mathbf{u}\in\mathbb{U}(t)\subset\mathbb{R}^m && \text{a.e. } t\in[t_0,t_f] \\
&\qquad \mathbf{x}(t)\in\mathbb{X} && \forall t\in[t_0,t_f] \\
&\qquad (\mathbf{x}(t_0),\mathbf{x}(t_f))\in\mathbb{X}_{01}
\end{aligned}
$$

Algorithm



Refinement strategy

The original coarse mesh is divided in K mesh intervals

$$\mathcal{S}_k = [\tau_{k-1},\tau_k[,\ k=1,\ldots,K-1 \quad \text{and} \quad \mathcal{S}_K = [\tau_{K-1},\tau_K]$$

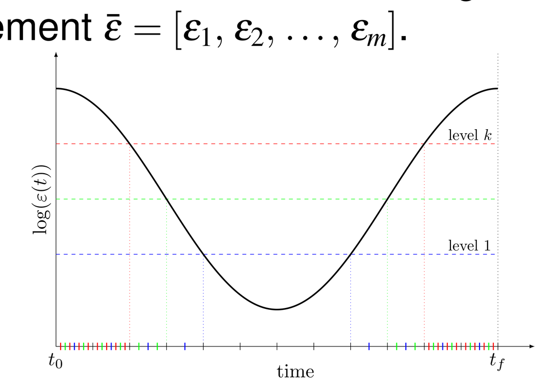
where $\tau_0 < \tau_1 < \ldots < \tau_K$ coincide with nodes.

The mesh intervals $\mathcal{S}_k$ form a partition of the time interval,

$$\bigcup_{k=1}^{K} \mathcal{S}_k = [t_0,t_f] \quad \text{and} \quad \bigcap_{k=1}^{K} \mathcal{S}_k = \emptyset.$$

After selecting the intervals $\mathcal{S}_k$ that verify the refinement criteria, they are divided into smaller subintervals according to the user–defined levels of refinement $\bar{\varepsilon} = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m]$.

A subinterval $\mathcal{S}_{k,i}$ is at the i^{th} level of refinement if $\mathcal{S}_{k,i} \in [\varepsilon_i, \varepsilon_{i+1}[$, for $i=1,\ldots,m$, and it will be refined by adding $\mathcal{N}^i$ of equidistant nodes between each two mesh points.



In our case, we define 3 refinement criteria:

- relative error of the trajectory (state variables) ($\varepsilon_{\mathbf{x}}$)
- relative error of the adjoint multipliers ($\mathbf{q}$ multipliers) ($\varepsilon_{\mathbf{q}}$)
- a combination of both criteria

and we consider a threshold for the relative error of the trajectory as the stopping criterion.

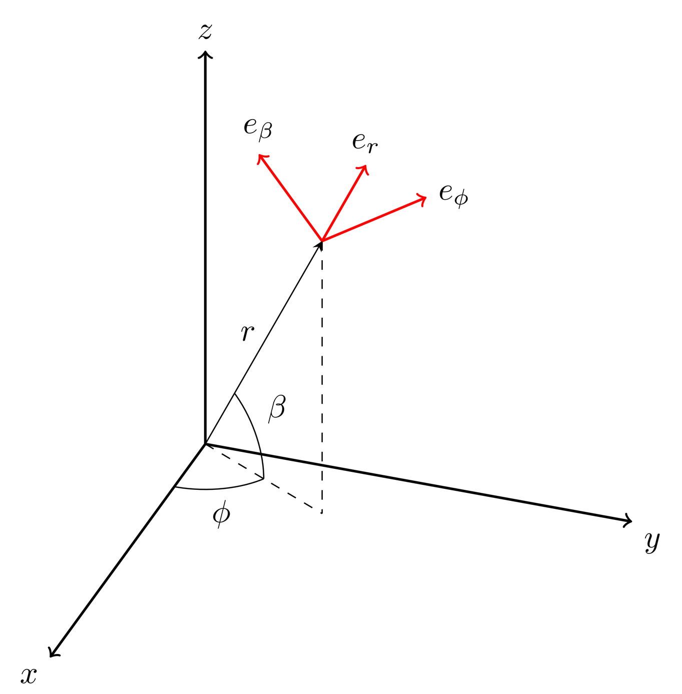
Relationship between multipliers: OCP and NLP

$$
\begin{array}{lllll}
 & \nearrow \text{Max. Principle} & \rightarrow q_{MP} & & \\
\text{Continuous OCP} \rightarrow & \text{Discrete OCP} & \rightarrow \text{Discrete MP} & \rightarrow q_{DMP} & \\
 & \searrow \text{Transcribe NLP} & \rightarrow \text{KT conditions} & \rightarrow q_{KT} &
\end{array}
$$

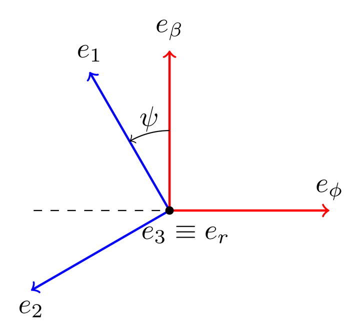
- q_{DMP} and q_{KT} coincide
- q_{DMP} is a discrete aproximation of q_{MP}
- q_{KT} is an output of the NLP solver

Kite power system

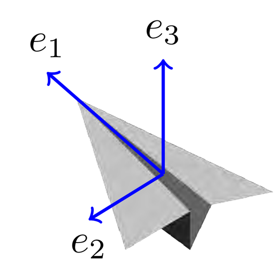
Coordinate Systems and Forces



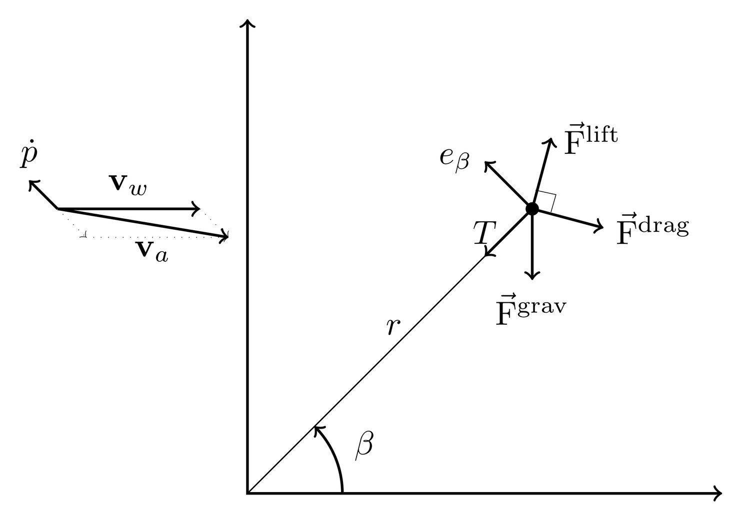
Global,Local



Local,Body



Body



Forces

Optimal Control Problem

Consider $\mathbf{x} = \left(r,\phi,\beta,\psi,\dot{\phi},\dot{\beta}\right)$ and $\mathbf{u} = (v_t,\alpha,\omega)$:

$$
\begin{aligned}
&\text{Maximize} \int_0^{t_f} \dot{r}(-T)\,\mathrm{d}t \\
&\text{subject to} \\
&\qquad \dot{\mathbf{x}}(t) = f(\mathbf{x}(t),\mathbf{u}(t)) && \text{a.e. } t\in[0,t_f] \\
&\qquad \mathbf{x}(0) = \mathbf{x}_0 \\
&\qquad \mathbf{x}_{f,\min} \le \mathbf{x}(t_f) \le \mathbf{x}_{f,\max} \\
&\qquad \mathbf{x}_{\min} \le \mathbf{x}(t) \le \mathbf{x}_{\max} && \forall t\in[0,t_f] \\
&\qquad \mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max} && \text{a.e. } t\in[0,t_f]
\end{aligned}
$$

where

$$
f(\mathbf{x}(t),\mathbf{u}(t)) = \begin{bmatrix} v_t \\ \dot{\phi} \\ \dot{\beta} \\ \omega \\ (F_\phi^{aer}(\alpha)+F_\phi^{iner})/(mr\cos(\beta)) \\ (F_\beta^{aer}(\alpha)+F_\beta^{grav}+F_\beta^{iner})/(mr) \end{bmatrix},
$$

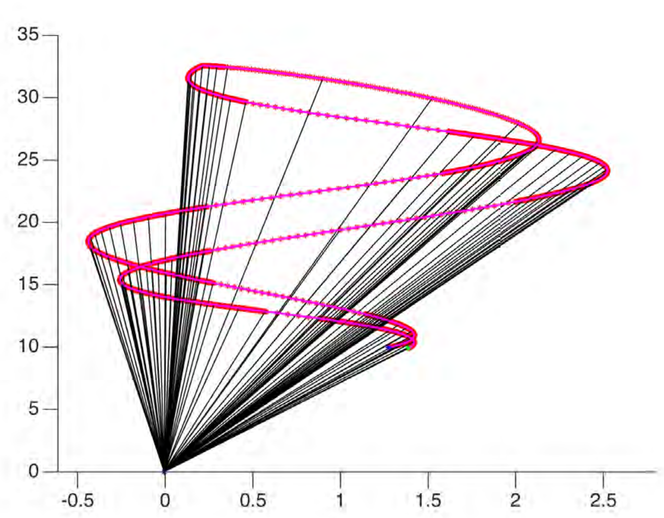
$$-T = F_r^{aer}(\alpha) + F_r^{grav} + F_r^{iner}.$$

Numerical results

The proposed algorithm is implemented in MATLAB using ICLOCS and IPOPT. The problem is solved using two meshes:

- π_{ML} mesh generate by the adaptive refinement strategy
- π_{F} equidistant spacing mesh considering the lowest Δt of π_{ML}

2D Problem

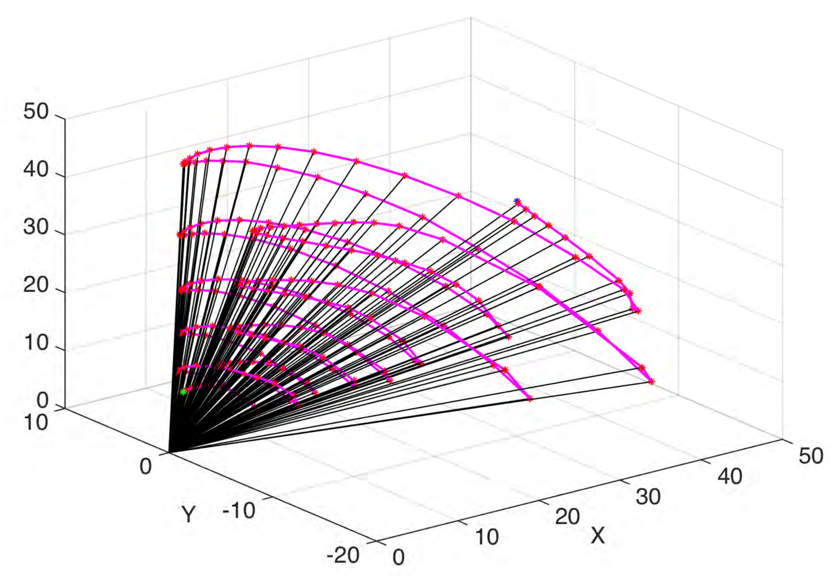


2D Optimal Trajectory

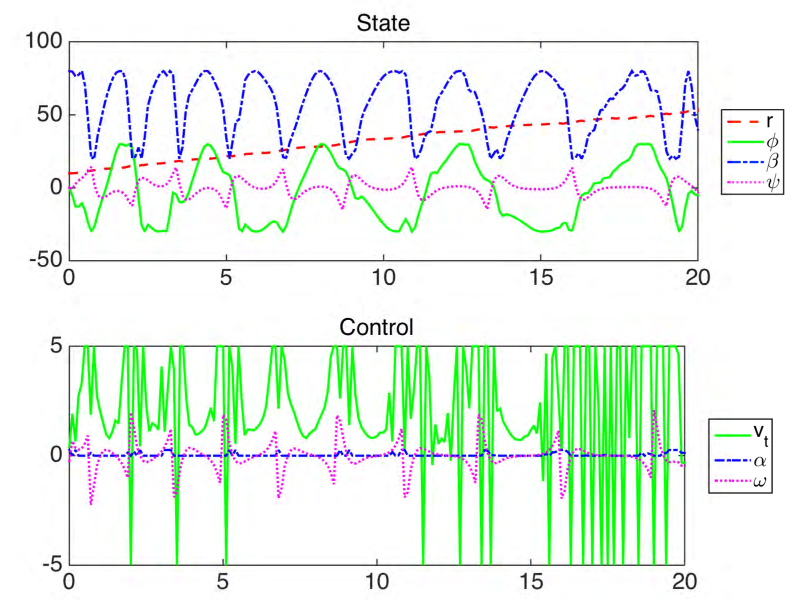
π_j	N_j	Δt_j	I_j	Objective	$\left\|\left\|\varepsilon_{\mathbf{x}}^{(j)}\right\|\right\|_\infty$	CPU time (s) Solve	CPU time (s) $\varepsilon_{\mathbf{x}}$
π_0	51	$^1/_{50}$	253	53.216968	1.795E^{-2}	4.754	0.107
π_1	474	$^1/_{500}$	122	54.156063	2.328E^{-3}	4.722	0.856
π_2	2877	$^1/_{5000}$	26	54.266839	3.168E^{-4}	6.107	3.383
π_{ML}	2877	$^1/_{5000}$	401	54.266839	3.168E^{-4}	15.583	4.346
π_{F}	5001	$^1/_{5000}$	1342	54.266873	2.201E^{-4}	431.234	10.913

Mesh comparison

3D Problem



Optimal Trajectory



State and Control

Concluding remarks

Main advantages of refinement strategies

- no need to define *a priori* the most appropriately mesh,
- local mesh resolution only where it's required,
- first solution obtained after few iterations since an initial coarse mesh is used,
- warm–start after refinement leads to faster re–solve,
- less nodes in the overall procedure results in significant savings in memory and computational cost.

... of our refinement algorithm

- multi–level refinement leads to faster convergence,
- local error on $\mathbf{q}$ gives a more accurate/faster error estimate,
- a solution is obtained very quickly (if the procedure ends in an early stage).

... in kite power systems

- much faster in the 2D problem,
- optimal solution found without informative initial guess in the 3D problem.

References

[1] Uwe Ahrens, Moritz Diehl, and Roland Schmehl, editors. *Airborne Wind Energy*. Green Energy and Technology. Springer Berlin Heidelberg, Berlin, Heidelberg, 2013.

[2] John T. Betts. *Practical methods for optimal control using nonlinear programming*. SIAM, 2001.

[3] Luís Tiago Paiva and Fernando A.C.C. Fontes. Adaptive time-mesh refinement in optimal control problems with state constraints. *Discrete and Continuous Dynamical Systems*, 35(9):4553–4572, April 2015.

[4] Richard B. Vinter. *Optimal Control*. Springer, 2000.

Acknowledgements

Support of FCT – Fundação para a Ciência e Tecnologia – under Grants PTDC/EEA-CRO/116014/2009 and PTDC/EEI-AUT/1450/2012.

U.PORTO FEUP FACULDADE DE ENGENHARIA UNIVERSIDADE DO PORTO

FCT Fundação para a Ciência e a Tecnologia

FCOMP-01-0124-FEDER-028894 FCT PTDC/EEI-AUT/1450/2012