Structural Modeling of Thin Membranes for Wind Energy Systems

Rijk de Rooij,[@,*] Mostafa Abdalla[*]

[@]rderooij@stanford.edu, [*]Aerospace Structures and Computational Mechanics, Delft University of Technology, The Netherlands

Introduction

Membrane structures have a rich history of use across many disciplines and are widely used in aerospace and structural engineering applications. More recently, thin membranes are also often applied in 'lighter-than-air' wind energy systems, see e.g. Fig 1. These membranes are especially attractive to airborne wind energy systems for their low mass to surface ratio and their ability to take complex shapes.

Figure 1: The KitePower is an example of a wind energy system concept that consists of a thin membrane structure [2].

Although membranes can carry tensile loads very well, they tend to wrinkle under the slightest compressive load. These wrinkles affect the structural performance of the entire airborne system. However, to model the structural behavior of wrinkles in a membrane, the mesh size in e.g. a traditional finite element model needs to be at least as small as the wrinkles to detect them, which often results in unacceptably high computational costs. Here, we present a validated method that allows us to determine the stress distribution in membranes using a mesh size that is much bigger than the individual wrinkles, and therefore leads to great computational savings [1].

Method

We modeled the membrane wrinkles as a continuous in-plane contraction of the membrane, using an interior-point implementation. This procedure yields one governing equation in addition to the standard equations:

$$\hat{\boldsymbol{E}}\hat{\boldsymbol{S}} = \mu \boldsymbol{I} \quad (1)$$

with $\hat{\boldsymbol{S}} > \boldsymbol{0}$ as the membrane stress, $\hat{\boldsymbol{E}} > \boldsymbol{0}$ as the wrinkling strain and $\mu \to 0_+$ as the penalty parameter. After using the finite element method to solve the equations for the membrane stresses, the membrane states are identified using (1) as:

- taut when $\hat{\boldsymbol{E}} \approx \boldsymbol{0}$;
- slack when $\hat{\boldsymbol{S}} \approx \boldsymbol{0}$;
- wrinkled when $\hat{\boldsymbol{S}} > \boldsymbol{0}$ and $\hat{\boldsymbol{E}} > \boldsymbol{0}$ are (almost) rank-one with perpendicular principal directions.

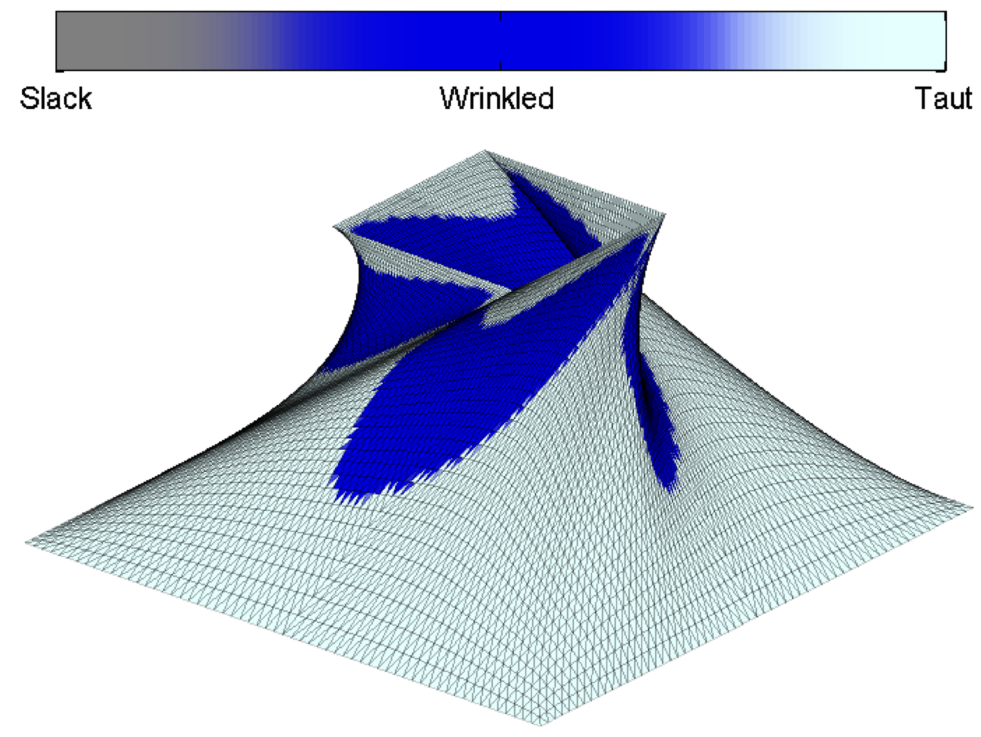
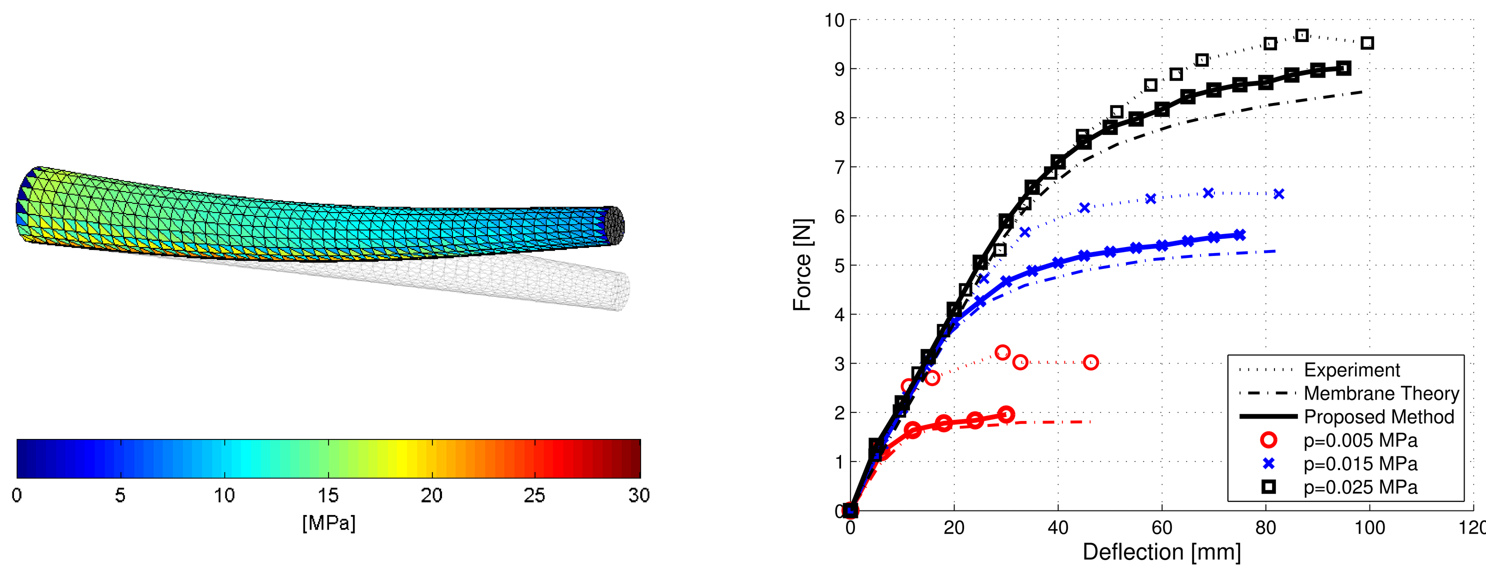


Figure 2: Membrane states in highly deformed annular square. The accurate representation of the membrane states was used for numerical verification.

We developed a mathematical proof for the robustness of the proposed method and performed several numerical examples to assess the efficiency and validation of this method.

Results

Fig. 2 shows the membrane states in an annular square with its inner edge pulled out and rotated over $90°$. The complex pattern in Fig. 2 was used to verify the proposed method. We then validated the method with experimental data using an inflated beam under lateral tip loading, see Fig. 3. Our method accurately captures the structural softening of the beam due to initiation of membrane wrinkling.



(a) (b)

Figure 3: (a) Contours of first principal stresses in a pressurized tapered beam with applied tip deflection, and (b) validation of the load-displacement curves with experimental data [3].

References

[1] de Rooij, R., and Abdalla, M. A finite element interior-point implementation of tension field theory. *Computers & Structures 151*, 0 (2015), 30 – 41.

[2] KitePower, 2015.

[3] Veldman, S. *Design And Analysis Methodologies for Inflated Beams*. DUP Science, 2005.